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ATTITUDE RESTORATION OF A MISSILE WITH THE AID OF THREE  
MAGNETOMETERS AND AN EMITTING ELECTRIC DIPOLE

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ATTITUDE RESTORATION OF A MISSILE WITH THE AID OF THREE  
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SUMMARY

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The simultaneous measurement of the Earth's magnetic field and of the electromagnetic field of a wave, emitted by a dipole placed on the missile, allows to determine unambiguously and at any chosen time the latter's attitude.

*author*

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A method of determination of the attitude of a rocket utilizing three magnetometers and a solar cell [1] has already been proposed. The interest of that method is in the simplicity of the optical system used. Unfortunately, its use is not general, for it imposes the presence of the Sun or of the Moon. It is thus interesting to liberate ourselves from such constraints. We now propose a method utilizing three magnetometers and the knowledge of the electric field emitted by a dipole placed in the axis of one of the magnetometers.

The three magnetic collectors, placed on board of the rocket, define three orthogonal directions  $(O, x, y, z)$  linked with the rocket, and allow measurement of the intensity of the Earth's magnetic field components along these three axes. If we take for reference the trihedron  $(O, X_n, Y_n, Z_n)$ ,  $Z_n$  bearing the magnetic field vector, but oriented in the opposite direction, and  $X_n$  being directed along the magnetic East in the

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\* Restitution de l'attitude d'un engin à l'aide de trois magnétomètres et d'un dipôle émetteur.

horizontal plane, the magnetic collectors allow the measurement of the Euler angles  $\theta_0$  and  $\varphi_0$ , but ignore the angle  $\psi_0$  which is necessary for an unambiguous determination of the position of the trihedron linked with the rocket (Fig. 1). The method proposed here has for aim the measurement of the angle  $\psi_0$ .

We propose to analyze the signals emitted by a dipole, placed on board of the rocket, using a system of antennas installed on the ground and consisting of three orthogonal dipoles of which the three directions overlap with the axes of the reference trihedron  $(O, X_R, Y_R, Z_R)$ ,  $Z_R$  being vertical and  $X_R$  overlapping with the earlier defined direction of  $X_H$ . The voltage measured at receivers' output depends upon the radiation pattern of the receiving antennas. These patterns are essentially function of the elevation and azimuth angles of the direction of propagation  $\Delta$ . These angles are obtained with a rather good approximation by trajectographic installations of the launching pads.

We shall assume that the emitting dipole is placed on the axis  $\underline{y}$ . When the rocket spins, the emitting dipole scans the plane  $(x, y)$ . The electric vector  $E$  then describes at ground level an ellipse situated in the plane perpendicular to the axis  $\Delta$  passing by the point  $O$  (Fig. 2). We shall denote this plane by  $(E)$ . If bring the origin of the trihedron  $(x, y, z)$  linked with the missile, to the point  $O$ , the planes  $(E)$  and  $(xy)$  intersect on the axis  $MM'$ . Let us denote by  $\Phi$  the angle between the axes  $\Delta$  and  $\underline{z}$ , by  $\Theta$  the angle between the axes  $\Delta$  and  $\underline{y}$ . In the course of a complete rotation of the rocket the angle  $\Theta$  varies by

$$\Theta = \frac{\pi}{2} - \Phi \quad \text{à} \quad \Theta = \frac{\pi}{2} + \Phi$$

passing twice by the value  $\Theta = \pi/2$ , when it passes through  $M$  and  $M'$ . Assuming that the rocket's proper rotation is great relative to the precession movement of the axis of rotation, the angle  $\Phi$  is given by

$$\cos \Phi = \frac{E_{\min}}{E_{\max}}, \quad (1)$$

$E_{\min}$  and  $E_{\max}$  being respectively the consecutive minima and maxima of the function representing the variation, as a function of time, of the intensity

of the electric vector received on the ground

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2}, \quad (2)$$

where  $E_x$ ,  $E_y$  and  $E_z$  are the values of the component of  $E$ , measured on each reception dipole, taking into account the radiation patterns.

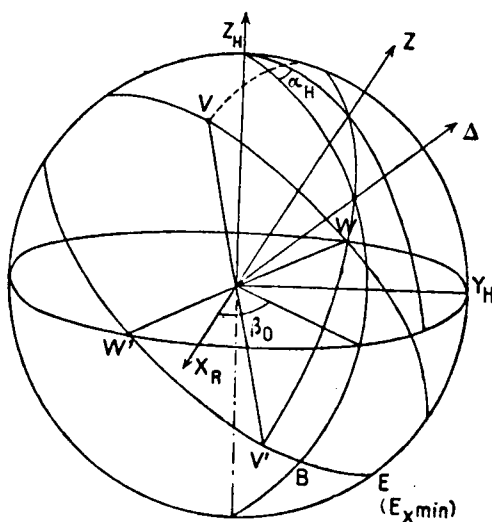


Fig. 1.

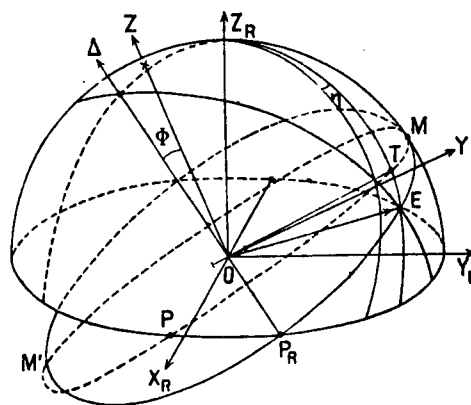


Fig. 2.

A first method, called the reference plane method, consists in measuring the angle  $\phi$  by formula (1), then in calculating  $\alpha_H$  by resolving the spherical triangle  $(Z_H, \Delta, z)$  (Fig. 1); this gives, in its turn, the Euler angle  $\psi_0$  searched for, by the relation

$$\psi_0 = (\beta_0 \pm \alpha_H) + \frac{\pi}{2}, \quad (3)$$

$\beta_0$  being the azimuth of the axis  $\Delta$  in the trihedron  $(O, X_R, Y_R, Z_R)$ . We may see from this formula that there is ambiguity as regards the value of the angle  $\psi_0$  and, by way of consequence, also as regards the value of the azimuth of the axis  $z$ . This stems from the fact, that during calculations of the angle  $\alpha_H$  we are ignorant of its sign and, consequently, that the plane defined by the axes  $(Z_H, z)$  may be situated on either side of the reference plane  $(Z_H, \Delta)$ , thus defining two possible positions for the axis of the missile, symmetrical relative to the plane  $(Z_H, \Delta)$ . In order to overcome this, it is sufficient in the last analysis, to know the sign of  $\text{tg}\theta$ .

A second method, said to be method of minima, allows to know the sign of  $\text{tg}\theta$ . This method consists in defining, at a given moment of time, the vertical plane containing the axis  $y$  of the emitting dipole. If, as was assumed, the axis  $y$  bears one of the two magnetometers of the right-hand section of the missile, the knowledge of the vertical plane containing this magnetometer allows then to compute the Euler angles defining the position of the trihedron linked with the missile.

The calculations can be simplified by choosing as the vertical plane that of the magnetic meridian. We then select the moments of time when the electric vector  $E$  is in that plane. These moments of time are characterized by the fact that the signal  $E_x$ , given by the dipole  $X_R$ , is minimum or zero. We then define a fictitious axis traversing the magnetic meridian plane simultaneously with the vector  $E$  (thus situated in the same vertical plane as the vector  $E$ ); let  $OT$  be that axis (Fig. 2). We denote by  $\varphi$  the angle between  $y$  and that fictitious axis in the right-hand section of the missile. The axis  $OT$  is defined in the latter plane as of the line of nodes in the trihedron  $(O, X_n, Y_n, Z_n)$  by the angle

$$\varphi_0 = \varphi_0 \pm \epsilon. \quad (4)$$

The sign to be chosen in this formula also depends on the sign of  $\text{tg}\theta$ .

We then compute in the magnetic meridian plane:

$$(Y_R, E) = L_R \quad \text{by} \quad \text{tg} L_R = \frac{|\sin \beta|}{\text{tg} \theta}.$$

Formula (4) provides two possible positions for the point  $T$  in the magnetic meridian plane, say  $T_1$  and  $T_2$ . But, since there is only one possible position for the vector  $E$  in this same plane, the position of this vector relative to points  $T_1$  and  $T_2$  provides immediately the sign of  $\text{tg}\theta$ .

Indeed:

- if  $E$  is between  $Z_R$  and  $T_1, T_2$ , we have  $\theta > \pi/2$  and  $\text{tg}\theta < 0$ ;
- if, to the contrary,  $T_1, T_2$  are between  $Z_R$  and  $E$ , we have  $\theta < \pi/2$  and  $\text{tg}\theta > 0$ .

We see that it is thus possible to know without ambiguity the value

of the angle  $\psi_0$ ; the knowledge of it will allow us to know the position of the trihedron linked with the rocket relative to the fixed trihedron  $(O, X_{II}, Y_{II}, Z_{II})$ .

Note that in the particular case (that often occurs in normal rocket launchings) when the angle  $\Phi$  is zero or very small, the angle  $\Theta$  is zero, just as  $\nu$  is. Then there is no angular dephasing between the vertical planes containing the electric vector and the emitting dipole. Thus, one is certain, that when  $E_X$  is minimum, the emitting dipole is the magnetic meridian plane. Formula (4) does not then present any ambiguity and the solution of the problem is very simple.

Now it remains to take into account the error engendered by the crossing ionized layers of the ionosphere by an electromagnetic wave. It is well known that the presence of such a medium induces the rotation of the electric vector in the plane of the wave (Faraday effect). The first method of calculation of the angle  $\Phi$  is not affected by the Faraday effect since the measurements are relative. The second method requires an approximate evaluation of the Faraday effect.

At any rate it is possible to measure the Faraday effect with a fairly great precision with the help of the measurement of the differential Doppler effect obtained practically by the emission of the harmonic of the central frequency.

This method of restoring the attitude of rockets offers the following interest:

- generality of use;
- the additional radioelectric equipment placed in the rocket is simple and makes no use of telemetry;
- the receiving device on the ground is installed definitely.

This method provides, moreover, the possibility of determining the content of electrons in the ionosphere between the altitude of the rocket and the ground.

\*\*\* THE END \*\*\*

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on 20 September 1965

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